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10MAT41

Fourth Semester B.E. Degree Examination, Dec.2015/Jan. 2016
Engineering Mathematics - IV
Time: 3 hrs .

## Note: 1. Answer FIVE full questions, selecting at least TWO questions from each part. <br> 2. Use of statistical tables is permitted.

## PART - A

1 a. Using Taylor series method, solve the problem $\frac{d y}{d x}=x^{2} y-1, y(0)=1$ at the point $x=0.2$. Consider unto $4^{\text {th }}$ degree terms.
(06 Marks)
b. Using R.K. method of order 4 , solve $\frac{d y}{d x}=3 x+\frac{y}{2}, y(0)=1$ at the points $x=0.1$ and $x=0.2$ by taking step length $\mathrm{h}=0.1$.
(07 Marks)
c. Given that $\frac{d y}{d x}=x-y^{2}, y(0)=0, y(0.2)=0.02, y(0.4)=0.0795, y(0.6)=0.1762$. Compute y at $\mathrm{x}=0.8$ by Adams-Bashforth predictor-corrector method. Use the corrector formula twice.
(07 Marks)
2 a. Evaluate $y$ and $z$ at $x=0.1$ from the Picard second approximation to the solution of the following system of equations given by $y=1$ and $z=0.5$ at $x=0$ initially.

$$
\frac{d y}{d x}=z, \quad \frac{d z}{d x}=x^{3}(y+z)
$$

(06 Marks)
b. Given $y^{\prime \prime}-x y^{\prime}-y=0$ with the initial conditions $y(0)=1, y^{\prime}(0)=0$. Compute $y(0.2)$ and $\mathrm{y}^{\prime}(0.2)$ by taking $\mathrm{h}=0.2$ and using fourth order Runge-Kutta method.
(07 Marks)
c. Applying Milne's method compute $y(0.8)$. Given that $y$ satisfies the equation $y^{\prime \prime}=2 \mathrm{yy}^{\prime}$ and y and $\mathrm{y}^{\prime}$ are governed by the following values. $\mathrm{y}(0)=0, \mathrm{y}(0.2)=0.2027, \mathrm{y}(0.4)=0.4228$, $y(0.6)=0.6841, y^{\prime}(0)=1, y^{\prime}(0.2)=1.041, y^{\prime}(0.4)=1.179, y^{\prime}(0.6)=1.468$. (Apply corrector only once).
(07 Marks)
3 a. Derive Cauchy Riemann equations in Cartesian form.
(06 Marks)
b. Find an analytic function $f(z)=u+i v$. Given $u=x^{2}-y^{2}+\frac{x}{x^{2}+y^{2}}$.
(07 Marks)
c. If $f(z)$ is a regular function of $z$, show that $\left[\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right]|f(z)|^{2}=4\left|f^{\prime}(z)\right|^{2}$
(07 Marks)

4 a. Find the bilinear transformation that maps the points $\mathrm{z}=-1, \mathrm{i},-1$ onto the points $\mathrm{w}=1, \mathrm{i},-1$ respectively.
(06 Marks)
b. Find the region in the w-plane bounded by the lines $x=1, y=1, x+y=1$ under the transformation $w=z^{2}$. Indicate the region with sketches.
(07 Marks)
c. Evaluate $\int_{C} \frac{e^{2 z}}{(z+1)(z-2)} d z$ where c is the circle $|\mathrm{z}|=3$.
(07 Marks)

## PART - B

5 a. Solve the Laplaces equation in cylindrical polar coordinate system leading to Bessel differential equation.
(06 Marks)
b. If $\alpha$ and $\beta$ are two distinct roots of $J_{n}(x)=0$ then prove that $\int_{0}^{1} x J_{n}(\alpha x) J_{n}(\beta x) d x=0$ if $\alpha \neq \beta$.
(07 Marks)
c. Express the polynomial, $2 \mathrm{x}^{3}-\mathrm{x}^{2}-3 \mathrm{x}+2$ interms of Legendre polynomials.
(07 Marks)
6 a. State and prove addition theorem of probability.
(06 Marks)
b. Three students A, B, C write an entrance examination. Their chances of passing are $1 / 2,1 / 3,1 / 4$ respectively. Find the probability that,
i) Atleast one of them passes.
ii) All of them passes.
iii) Atleast two of them passes.
(07 Marks)
c. Three machines A, B, C produce respectively $60 \%, 30 \%, 10 \%$ of the total number of items of a factory. The percentages of defective outputs of these three machines are respectively $2 \%, 3 \%$ and $4 \%$. An item is selected at random and is found to be defective. Find the probability that the item was produced by machine C .
(07 Marks)
7 a. The pdf of a random variable x is given by the following table:

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{x})$ | k | 2 k | 3 k | 4 k | 3 k | 2 k | k |

Find: i) The value of $k$
ii) $\mathrm{P}(\mathrm{x}>1)$
iii) $\mathrm{P}(-1<\mathrm{x} \leq 2)$
iv) Mean of $x$
v) Standard deviation of $x$.
(06 Marks)
b. In a certain factory turning out razar blades there is a small probability of $1 / 500$ for any blade to be defective. The blades are supplied in packets of 10 . Use Poisson distribution to calculate the approximate number of packets containing, i) One defective, ii) Two defective, in a consignment of 10000 packets.
(07 Marks)
c. In a normal distribution $31 \%$ of items are under 45 and $8 \%$ of items are over 64 . Find the mean and standard deviation of the distribution.
(07 Marks)
8 a. A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 39350 kilometers with a standard deviation of 3260 . Can it be considered as a true random sample from a population with mean life of 40000 kilometers? (Use 0.05 level of significance) Establish $99 \%$ confidence limits within which the mean life of tyres expected to lie. (Given that $\mathrm{Z}_{0.05}=1.96, \mathrm{Z}_{0.01}=2.58$ )
(06 Marks)
b. Ten individuals are chosen at random from a population and their heights in inches are found to be $63,63,66,67,68,69,70,70,71,71$. Test the hypothesis that the mean height of the universe is 66 inches. (Given that $\mathrm{t}_{0.05}=2.262$ for 9 d.f.)
(07 Marks)
c. Fit a Poisson distribution to the following data and test the goodness of fit at $5 \%$ level of significance. Given that $\psi_{0.05}^{2}=7.815$ for 4 degrees of freedom.

| $x$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency | 122 | 60 | 15 | 2 | 1 |

(07 Marks)


Fourth Semester B.E. Degree Examination, Dec.2015/Jan. 2016 Graph Theory and Combinatorics

Time: 3 hrs.
Max. Marks: 100

## Note: Answer any FIVE full questions, selecting atleast TWO questions from each part.

## PART - A

1 a. Determine the order $|\mathrm{V}|$ of the graph $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ in the following cases
i) G is a cubic graph with 9 edges
ii) G is regular with 15 edges
iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3. (07 Marks)
b. Define isomorphism of any two graphs. Show that the following graphs are not isomorphic.
(06 Marks)


Fig.Q1(b)
c. Prove that : Any connected graph G is Euler if and only if all the vertices of $G$ are of even degree.
(07 Marks)

2 a. Define :
i) Planar graph
ii) Non - planar graph

Show that the complete graph $\mathrm{K}_{\mathrm{S}}$ is a non - planar graph.
(07 Marks)
b. Write down the steps involved in the detection of planarity by method of elementary reduction.
(06 Marks)
c. Determine chromatic number and chromatic polynomial for the graph given below :
(07 Marks)


Fig.Q2(c)

3 a. Prove that A connected graph is a tree if and only if it is minimally connected. (07 Marks)
b. Find all the spanning tress of the graph shown below :


Fig.Q3(c)
c. Obtain the optimal prefix code for the word VISVESVARAYA. Indicate the code.

4 a. Determine the shortest path from the vertex 'a' to every other vertices in the following directed graph(Fig.Q4(a)).
(08 Marks)

Fig.Q4(a)

Fig.Q4(b)

Fig.Q4(c)
b. Using the Kruskal's algorithm, find a minimal spanning tree of the weighted graph shown in Fig.Q4(b).
(06 Marks)
c. For the network shown in Fig.Q4(c), determine the maximum flow between the vertices A and $D$ by identifying the cut -set of minimum capacity.
(06 Marks)

## PART - B

5 a. A woman has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations?
i) Two particular persons will not attend separately
ii) Two particular persons will not attend together.
(07 Marks)
b. Determine the coefficient of:
i) $x y z^{2}$ in the expansion of $(2 x-y-z)^{4}$
ii) $x^{2} y^{2} z^{3}$ in the expansion of $(3 x-2 y-4 z)^{7}$.
(07 Marks)
c. Using the moves $\mathrm{R}(\mathrm{x}, \mathrm{y}) \rightarrow(\mathrm{x}+1, \mathrm{y})$ and $\mathrm{U}:(\mathrm{x}, \mathrm{y}) \rightarrow(\mathrm{x}, \mathrm{y}+1)$, find in how many ways can one go :
i) From $(0,0)$ to $(6,6)$ and not rise above the line $y=x$.
ii) From $(2,1)$ to $(7,6)$ and not rise above the line $y=x-1$.
iii) From $(3,3)$ to $(10,15)$ and not rise above the line $y=x+5$.
(06 Marks)
6 a. Determine the number of positive integers n such that $\mathrm{l} \leq \mathrm{n} \leq 100$ and n is not divisible by 2 , 3 , or 5 .
(07 Marks)
b. There are n pairs of Children's gloves in a box. Each pair is of a different colour. Suppose the right gloves are distributed at random to ' $n$ ' children and thereafter the left gloves are also distributed to them at random. Find the probability that :
i) no child gets a matching pair
ii) every child gets a matching pair
iii) exactly one child gets a matching pair and
iv) atleast 2 children get matching pairs.
(07 Marks)
c. Find the rook polynomial for the $3 \times 3$ board $y$ using the expansion formula.
(06 Marks)
7 a. Find the generating function for the following sequences :
i) $1^{2}, 2^{2}, 3^{2}, 4^{2}$
ii) $0^{2}, 1^{2}, 2^{2}, 3^{2}, 4^{2}, \cdots-\cdots$
ii) $1^{3}, 2^{3}, 3^{3}, 4^{3}$
vi) $0^{3}, 1^{3}, 2^{3}, 4^{3}$,
(07 Marks)
b. In how many ways can we distribute 24 pencils to 4 children so that each child gets atleast 3 pencils but not more than 8 .
c. Using generating function, find the number of partitions of $\mathrm{n}=6$.
(06 Marks)
a. A bank pays a certain $\%$ of annual interests on deposits, compounding the interests once in 3 months. If a deposit doubles in 6 years and 6 months, what is the annual $\%$ of interest paid by the bank?
(06 Marks)
b. Solve the recurrence relation $a_{n+2}-6 a_{n+1}+9 a_{n}=3 \times 2^{n}+7 \times 3^{n}$ for $n \geq 0$, given $a_{0}=1$, $a_{1}=4$.
(07 Marks)
c. Solve the recurrence relation $a_{n+1}-a_{n}=3^{n}, n \geq 0$ with $a_{0}=1$ by using method of generating function.
(07 Marks)


Fourth Semester B.E. Degree Examination, Dec.2015/Jan. 2016 Design and Analysis of Algorithms

Time: 3 hrs .
Max. Marks: 100

## Note: Answer any FIVE full questions, selecting atleast TWO questions from each part.

## PART - A

1 a. What is an algorithm? Explain the notion of algorithm with an example. (06 Marks)
b. Explain the asymptotic notations with examples.
(06 Marks)
c. Write an algorithm for selection sort. Analyze its efficiency.
(08 Marks)

2 a. What is divide and conquer? Explain the general method of divide and conquer. (06 Marks)
b. Write an algorithm for merge sort. Analyze its efficiency.
(08 Marks)
c. Apply quick sort on following list and draw recursive call tree : 5, 3, 1, 9, 8, 2, 4, 7 .
(06 Marks)

3 a. What is minimum cost spanning tree? Apply Prim's and Kruskal's algorithm on Fig. Q3(a).
(10 Marks)


Fig.Q3(a)
b. Write Dijkstra's shortest path algorithm. Apply Dijkstra's algorithm on Fig. Q3(b) to obtain shortest paths.
(10 Marks)


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4 a. Explain dynamic programming. Find transitive closure using Warshall's algorithm for the digraph Q4(a).
(06 Marks)


Fig.4Q(a)
b. Find all pair shortest paths using Floyd's algorithm for the graph Fig. Q4(b).
(08 Marks)


Fig.Q4(b)
c. Find the optimal solution for the following instance of knapsack problem using dynamic programming.
(06 Marks)

| Item | Weight | Value |
| :---: | :---: | :---: |
| 1 | 2 | 12 |
| 2 | 1 | 10 |
| 3 | 3 | 20 |
| 4 | 2 | 15 |

Capacity $\mathrm{W}=5$
PART-B

5 a. Explain different decrease and conquer approaches using example.
(06 Marks)
b. Differentiate between DFS and BFS.
(04 Marks)
c. Write Horspool's algorithm for string matching. Find the pattern : BARBER.

In the text : JIM_SAW_ME_IN_A_BARBERSHOP.
(10 Marks)
6 a. What is decision tree? Draw the decision tree for three element selection sort and estimate its lower bound.
(10 Marks)
b. Explain following with examples :
i) P problems
ii) NP problems
iii) NP - complete problems.
(10 Marks)
7 a. What is back tracking? Draw the state space tree for 4 - queen's problem.
(08 Marks)
b. What is branch and bound method? Apply branch and bound to the following instance of assignment problem :
(06 Marks)
\(\left[\begin{array}{cccc}Job 1 \& Job 2 \& Job 3 \& Job 4 <br>
9 \& 2 \& 7 \& 8 <br>
6 \& 4 \& 3 \& 7 <br>
5 \& 8 \& 1 \& 8 <br>

7 \& 6 \& 9 \& 4\end{array}\right]\)| Person a |
| :--- |
| Person b |
| Person c |
| Person d |

c. Explain approximation algorithm for traveling salesman problem.
(06 Marks)
8 a. What is PRAM? Explain PRAM algorithm with example.
(06 Marks)
b. Explain various computational models.
(06 Marks)
c. What is list ranking? Explain different types of list ranking.
(08 Marks)

Fourth Semester B.E. Degree Examination, Dec.2015/Jan. 2016 Unix and Shell Programming
Time: 3 hrs .
Max. Marks: 100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

## PART - A

1 a. Explain the Architecture of UNIX operating system with a neat diagram.
b. Describe the salient features of UNIX operating system.
(08 Marks)
c. Write a note on man command.

2 a. Explain the different types of files supported in UNIX.
(06 Marks)
b. Which command is used for listing file attributes? Explain significance of each field in the output.
(08 Marks)
c. Explain with a neat diagram the three modes of Vi - editor.
(06 Marks)
3 a. What are standard input, standard output and standard error? Explain in detail with example.
b. Define the term process. Explain the mechanism of process creation in UNIX.
(06 Marks)
c. Explain the following command with an example
i) Running jobs in background (\& and nohup)
ii) Execute later (at and batch)
(08 Marks)
4 a. Write a note on sort and find command.
(08 Marks)
b. Differentiate between Hard link and Soft link in UNIX with example.
(06 Marks)
c. Explain the following commands with example
i) Head
ii) tail
iii) $\operatorname{Pr}$
(06 Marks)

## PART - B

5 a. What is the difference between a wild card and regular expression? Explain 'grep' command using $n, l$ and $f$ option with example.
(06 Marks)
b. What are Extended Regular Expressions? Explain any four ERE set used by grep and egrep.
( $\mathbf{0 6}$ Marks)
c. Explain line addressing and context addressing in sed with example.
$\mathbf{( 0 8 ~ M a r k s )}$

6 a. What is shell programming? Write a shell program to create a menu which displays,
i) List of files
ii) Current date
iii) Process status
iv) Current user of the system and
v) Quit to UNIX
(08 Marks)
b. Explain shell features of 'while' and 'for' with syntax.
(06 Marks)
c. Explain the use of test and [ ] to evaluate an expression in shell.
(06 Marks)

7 a. What is AWK? Explain any three built - in function in AWK.
(06 Marks)
b. Write an AWK sequence to find HRA, DA and Netpay of an employee, where DA is $50 \%$ of basic, HRA is $12 \%$ of basic and the Netpay is the sum of HRA, DA and Basic pay.
c. Briefly describe built in variables in AWK.
(08 Marks)

8 a. Explain with example the string handling function supported by perl.
(06 Marks)
(08 Marks)
b. Explain Lists, Arrays and Associative Arrays with respect to perl.
(06 Marks)
c. Write a perl script to convert decimal number to binary number.
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# Fourth Semester B.E. Degree Examination, Dec.2015/Jan. 2016 Microprocessor 

Time: 3 hrs.
Max. Marks:100
Note: Answer any FIVE full questions, selecting atleast TWO questions from each part.

## PART - A

1 a. Draw and explain the programming model of 8086 through Pentium processors. ( $\mathbf{0 6}$ Marks)
b. Explain with neat block diagram the working principle of 8086 Architecture. ( 08 Marks)
c. Discuss the Flag registers of 8086 with examples.
(06 Marks)
2 a. Briefly explain the concept of Memory paging in 80386 microprocessor, with suitable schematic diagram.
(08 Marks)
b. Explain the execution of PUSH and POP Instruction, with respect to Stack Addressing mode.
c. Discuss the Importance of protected mode memory addressing.

3 a. Write 8086 ALP for Reverse a string and check is it palindrome. (06 Marks)
b. Explain the following Instructions with examples :
i) XLAT
ii) LEA
iii) CMP
iv) SAHF.
(08 Marks)
c. What are Assembler Directives? Explain any four directives with suitable examples.
(06 Marks)
4 a. Explain short, near and far jump instructions with examples.
(08 Marks)
b. Discuss the following instructions with examples :
i) SHR
ii) SHL
iii) $R C R$
iv) TEST.
(06 Marks)
c. Briefly explain the string comparison instructions.
(06 Marks)

## PART - B

5 a. Differentiate between Macros and Procedures.
(06 Marks)
b. Explain the basic rules for using Assembly language with $\mathrm{C} / \mathrm{C}++$ for 16 bit DOS applications with the help of examples.
(08 Marks)
c. What is Inline Assembly? Explain its need.
(06 Marks)
6 a. Explain the functions of following pins in 8086.
i) $\mathrm{MN} / \overline{\mathrm{MX}}$
ii) ALE
iii) $\overline{\mathrm{BHE}}$
iv) INTR.
(08 Marks)
b. With neat diagram, explain minimum mode of 8086 system.
(07 Marks)
c. Explain Bus timings for Read and Write operation for minimum mode of 8086 system.
(05 Marks)
7 a. Explain any two methods of Address decoding technique with schematic diagram.
(08 Marks)
b. Design an 8086 based system with the following specifications :
i) 8086 in Minimum mode
ii) 64 Kbyte EPROM
iii) 64 Kbyte RAM.

Draw the completer schematic diagram of the design Indicating memory map.
(08 Marks)
c. Differentiate between Memory mapped I/O and Direct I/O.
(04 Marks)
8 a. Explain with neat block diagram the working operation of 8255 PPI.
(08 Marks)
b. Discuss the basic DMA controller operation in Microprocessor system.
(06 Marks)
c. Explain any three types 8086 Interrupts.
(06 Marks)


Fourth Semester B.E. Degree Examination, Dec.2015/Jan. 2016

# Computer Organization 

Time: 3 hrs.
Max. Marks: 100

## Note: Answer FIVE full questions, selecting at least TWO questions from each part.

## PART - A

1 a. Explain the function of processor registers with a block diagram.
(08 Marks)
b. Derive the basic performance equation. Discuss the measures to improve the performance.
(08 Marks)
c. List the different systems used to represent of signed number and give one example for each.
(04 Marks)
2 a. What is an addressing mode? Explain only four addressing modes with an example for each.
( 10 Marks)
b. Registers $R_{1}$ and $R_{2}$ of computer, contain the decimal values 1200 and 4600 . What is EA of the memory opened in each of the following instructions?
i) Load $20\left(R_{1}\right), R_{5}$
ii) Move \# 3000, $R_{5}$
iii) Store $R_{5}, 30\left(R_{1}, R_{2}\right)$
iv) Add - $\left(R_{2}\right) R_{5}$
v) Subtract $\left(R_{1}\right)+, R_{5}$
(05 Marks)
c. What is subroutine linkage? Explain with an example subroutine linkage using linkage register.
(05 Marks)
3 a. What is interrupt? Explain polling and vectored interrupts.
(07 Marks)
b. What is bus arbitration? Explain the centralized arbitration with a neat diagram.
(07 Marks)
c. What is DMA? Explain the registers in a DMA interface.
(06 Marks)
4 a. Explain with block diagram a general 8 bit parallel interface.
(10 Marks)
b. Describe how a read operation is performed on the PCI bus.
(10 Marks)

## PART - B

5 a. Draw the organization of a $1 \mathrm{~K} \times 1$ memory cell and explain its working.
(08 Marks)
b. Show with diagram the memory hierarchy with respect to speed, size and cost.
(05 Marks)
c. With a block diagram explain about direct mapping cache memory.
(07 Marks)
6 a. Discuss the Booth's multiplication algorithm, with an example.
(10 Marks)
b. With figure, explain circuit arrangements for binary division.
(05 Marks)
c. Illustrate the steps for non - restoring division algorithm on the following data : dividend $=1000$, divisor $=11$.
(05 Marks)
7 a. List out the actions needed to execute the instruction add $\left(R_{3}\right), R_{1}$. Write and explain sequence of control steps for the execution of the same.
(08 Marks)
b. Write a control sequence for on unconditional branch instructions.
(04 Marks)
c. Explain the 3 bus organization of the processor.
(08 Marks)
8 a. With a neat diagram, explain the organization of a shared memory multiprocessor. (08 Marks)
b. What is hardware multithreading? Explain the different approaches to hardware multithreading.
(08 Marks)
c. Explain single instruction stream, multiple data stream (SIMD).
(04 Marks)
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## Fourth Semester B.E. Degree Examination, Dec.2015/Jan. 2016 Advanced Mathematics - II

Time: 3 hrs .

Max. Marks: 100

## Note: Answer any FIVE full questions.

1 a. Find the direction cosines of the line which is perpendicular to the lines with direction cosines ( $3,-1,1$ ) an ( $-3,2,4$ ).
(06 Marks)
b. If $\cos \alpha, \cos \beta, \cos \gamma$ are the direction cosines of a line, then prove the following:
i) $\sin ^{2} \alpha+\sin ^{2} \beta+\sin ^{2} \gamma=2$
ii) $\cos 2 \alpha+\cos 2 \beta+\cos 2 \gamma=-1$
(07 Marks)
c. Find the projection of the line AB on the line CD where $\mathrm{A}=(1,2,3), \mathrm{B}=(1,1,1)$, $\mathrm{C}=(0,0,1), \mathrm{D}=(2,3,0)$.
(07 Marks)
2 a. Find the equation of the plane through (1, -2, 2), ( $-3,1,-2$ ) and perpendicular to the plane $2 x-y-z+6=0$.
(06 Marks)
b. Find the image of the point $(1,-2,3)$ in the plane $2 x+y-z=5$.
(07 Marks)
c. Find the shortest distance between the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.
(07 Marks)

3 a. Find the constant ' $a$ ' so that the vectors $2 i-j+k, i+2 j-3 k$ and $3 i+a j+5 k$ are coplanar.
(06 Marks)
b. Prove that $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}]=2[\vec{a}, \vec{b}, \vec{c}]$.
(07 Marks)
c. Find the unit normal vector to both the vectors $4 i-j+3 k$ and $-2 i+j-2 k$. Find also the sine of the angle between them.
(07 Marks)
4 a. A particle moves along the curve $x=t^{3}+1, y=t^{2}, z=2 t+5$ where $t$ is the time. Find the components of its velocity and acceleration at time $t=1$ in the direction of $2 i+3 j+6 k$.
(06 Marks)
b. Find the angle between the surfaces $x^{2}+y^{2}+z^{2}=9$ and $x=z^{2}+y^{2}-3$ at the point (2, -1, 2).
(07 Marks)
c. Find the directional derivative of $\phi=x y^{2}+y z^{3}$ at the point $(1,-2,-1)$ in the direction of the normal to the surface $x \log z-y^{2}=-4$ at $(-1,2,1)$.
(07 Marks)

5 a. Prove that $\operatorname{div}(\operatorname{curl} \overrightarrow{\mathrm{A}})=0$.
(06 Marks)
b. Find div $\vec{F}$ and $\operatorname{curl} \overrightarrow{\mathrm{F}}$ where $\overrightarrow{\mathrm{F}}=\nabla\left(\mathrm{x}^{3}+\mathrm{y}^{3}+\mathrm{z}^{3}-3 \mathrm{xyz}\right)$.
(07 Marks)
c. Show that the vector $\vec{F}=\left(3 x^{2}-2 y z\right) i+\left(3 y^{2}-2 z x\right) j+\left(3 z^{2}-2 x y\right) k$ is irrotational and find $\phi$ such that $\vec{F}=\operatorname{grad} \phi$.
(07 Marks)

6 a. Find: $L\{\cos t \cos 2 t \cos 3 t\}$.
b. Find: i) $L\left\{e^{-t} \cos ^{2} t\right\}$, ii) $L\left\{t e^{-t} \sin 3 t\right\}$.
(07 Marks)
c. Find: $L\left\{\frac{\cos a t-\cos b t}{t}\right\}$.

7 a. Find: $L^{-1}\left\{\frac{4 s+5}{(s-1)^{2}(s+2)}\right\}$.
(06 Marks)
b. Find: i) $L^{-1}\left\{\frac{s+2}{s^{2}-4 s+13}\right\}$,
ii) $\mathrm{L}^{-1}\left\{\log \left(\frac{\mathrm{~s}+\mathrm{a}}{\mathrm{s}+\mathrm{b}}\right)\right\}$.
(07 Marks)
c. Find: $L^{-1}\left\{\frac{1}{s^{2}(s+1)}\right\}$.
(07 Marks)

8 a. Using Laplace transforms, solve $\frac{d^{2} y}{d x^{2}}-2 \frac{d y}{d x}+y=e^{2 t} \quad$ with $y(0)=0, y^{\prime}(0)=1 . \quad$ (10 Marks)
b. Using Laplace transformation method solve the differential equation $y^{\prime \prime}+2 y^{\prime}-3 y=\sin t$, $y(0)=y^{\prime}(0)=0$.

